## Experiment 6
## Moment of inertia and angular acceleration

### *Related topics*

Angular velocity, rotary motion, moment, moment of inertia of a disc, moment of inertia of a bar, moment of inertia of a mass point.

### *Task*

A moment acts on a body which can be rotated about a bearing without friction. The moment of inertia is determined from the angular acceleration.

### *Problems*

From the angular acceleration, the moment of inertia are determined as a function of the mass and of the distance from the axis of rotation.

1. of a disc,

2. of a bar,

3. of a mass point.

### *Theory*

The relationship between the angular momentum $\vec{L}$ of a rigid body in the stationary coordinate system with its origin at the centre of gravity, and the moment $\vec{\tau}$ acting on it, is

$$\vec{\tau} = \frac{d}{dt}\vec{L} \tag{1}$$

The angular momentum is expressed by the angular velocity v R and the inertia tensor I from

$$\vec{L} = I.\vec{w}$$

that is, the reduction of the tensor with the vector.

In the present case, $\vec{w}$ has the direction of a principal inertia axis (Z-axis), so that L has only one component:

$$L_z = I_z.w$$

Where $I_z$ is the Z-component of the principal inertia tensor of the body. For this case, equation (1) reads

$$\tau_z = I_z.\frac{dw}{dt}$$

The moment of force F:

$$\vec{\tau} = \vec{r} \; x \; \vec{F}$$

and, for $\vec{r} \perp \vec{F}$
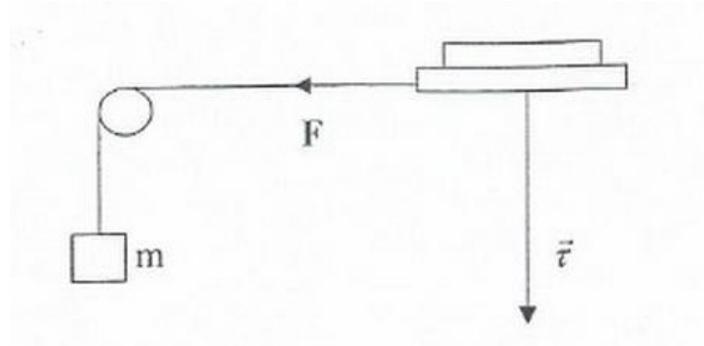
$$\vec{\tau} = \vec{r} \; x \; m.\vec{g}$$



Figure 1: Moment of gravity force effect on rotating disk

Finally, the equation of motion:

$$m.g.r = I_z.\frac{dw}{dt} = I_z.\alpha$$

From this, we get:

$$\frac{m.g.r}{\alpha} = I_{z-experimental}$$

If the system comprised of more than one object, moment of inertia of the system equals to the sum of the moment of inertia of each object.

$$I_{system} = \Sigma \, I_z$$

## Set-up and Procedure

The experimental set-up is arranged as shown in Fig. 1. The rotary bearing, with the blower switched on, is aligned horizontally with the adjusting feet on the tripod. The release trip must be so adjusted that it is in contact with the inserted sector mark in the set condition. The precision pully is clamped so that the thread floats horizontally above the rotating plane and is aligned with the pulley.

While using the bar, the fork type light barrier is positioned in such a way that, the end of the bar, lying opposite to the angular screen standing just in front of the light path, is held by the holder.

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While using the turntable, before the beginning of the experiment, the pin of the holder through the bore hole near the edge fixes the turntable. The fork type light barrier is positioned such that, the screen connected to the turntable, is in front of the light ray.

Whenever the holder is released, the light barrier must be interrupted at that moment.

The measurement of angular acceleration is done in the following manner:

-The light barrier key must be set at "⌐⌐⌐" and the "Reset" button is pressed.

- The time 't' indicated is the time for the acceleration. According to

$$\frac{1}{2}.\alpha.t^2 = \varphi$$

the acceleration is obtained. $\varphi: angular\ displacement$

### *Experiment and Results*

Find the moment of inertia of the disk and metal bar using equation (2). Result of this equation gives the experimental moment of inertia of each object.

Find the theoretical moment of inertia using equations below and compare with experimental results.

   1- For the disk with radius, r and mass, m.

$$\frac{1}{2}m_d.r_d^2 = I_{z-disk}$$

From the data for the disk:     $r_d = 0.175$ m, $m_d = 0,829$ kg

   2- For the metal rod with length, L and mass, m.

$$\frac{1}{12}m_r.L_r^2 = I_{z-rod}$$

From the data for the rod:     $m_r = 0,158\ kg,\ L_r = 0,730\ m$

   3- For a mass point of mass m at a distance r from the axis of rotation, one obtains;

$$m_p.r_p^2 = I_{z-point\ mass}$$

**1-) Determine moment of inertias of the disk and rod and fill in the table. Determine both experimental and theoretical results and compare them. Find the %error.**

**Table 1. Experimental and theoretical moment of inertias of disk and metal bar**

|  | *t(s)* | $I_{z-theoric}$ | $I_{z-experimental}$ |
|---|---|---|---|
| *Disk* |  |  |  |
| *Metal Rod* |  |  |  |

$$\%error = \frac{I_{experimental} - I_{theorical}}{I_{theorical}} x100 =$$

**2-) Calculate and fill in the table for different masses used in your experiment**

Table 2. The change of moment of inertia of point mass with its mass

| $m_p(kg)$ | $t(s)$ | $I_{z-rod}$ | $I_{z-point\ mass}$ | $I_{z-system-theoric}$ | $I_{z-experimental}$ |
|-----------|--------|-------------|---------------------|------------------------|----------------------|
|           |        |             |                     |                        |                      |
|           |        |             |                     |                        |                      |
|           |        |             |                     |                        |                      |

$I_z(kg/m^2)$

$r_p(m)$

**3-) Calculate and fill in the table for distances used in your experiment**

**Table 3. The change of moment of inertia of point mass with the distance from rotation axis**

| $r_p(m)$ | $t(s)$ | $I_{z-rod}$ | $I_{z-point\ mass}$ | $I_{z-system-theoric}$ | $I_{z-experimental}$ |
|---|---|---|---|---|---|
| | | | | | |
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## *Discuss the results*

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