EXPERIMENT 7
PROJECTILE MOTION

1. Objectives

- To learn the fundamentals of projectile(launching) motion with the different angles.
- To determine the range as a function of the angle of inclination.
- To learn the motion relations of both height for the vertical motion and the range for the horizontal motion with the different projector (throwing) angles.
- To determine the maximum height of projection as a function of the angle of inclination.
- To determine the maximum range as a function of the initial velocity.
- Related topics are trajectory parabola, motion involving uniform acceleration, ballistics.

2. Equipment

Ballistic unit (the unit launching the steel ball)
Recording paper, 1 roll, 25 m
Steel ball, d = 19 mm
Two-tier platform support
Meter scale, demo, l = 1000 mm
Barrel base (launching place)
Speed measuring attachement
Power supply 5 VDC/2.4 A

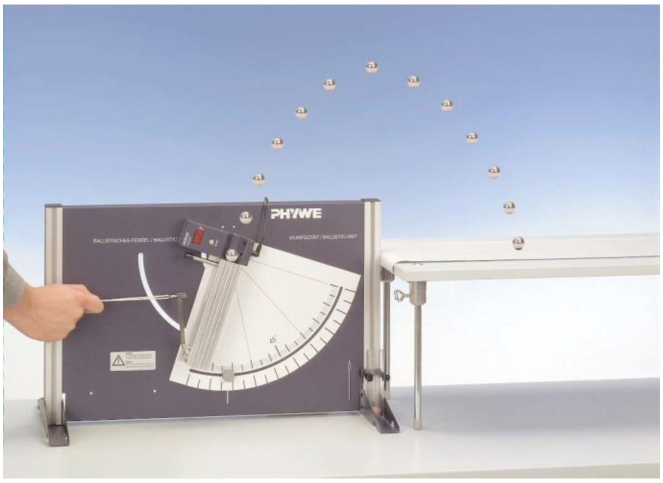

Figure 1 Experimental set-up for measuring the maximum range of a projectile with additional equipment to measure the initial velocity.

- The ballistic unit and barrel base which are used for launching the steel ball have three speed levels from the lowest to the highest speed, respectively. The angle of motion can be adjusted by changing the barrel base.
- The recording paper is used for the determination of the point where the steel ball hits. It also represents the maximum range point for the related motion.
- The speed measuring device which is shown by red digital segment depicts(shows) the launching speed of ball.
- The meter scale which has the scale of 1000mm=1m is used in order to measure both maximum height in vertical direction and maximum range in horizontal direction.

3. Theory

The system can be investigated on two directions of x and y. Just as the horizontal motion is represented by the function of x, the vertical motion is represented by the function of y in this system. This situation is given in Fig. 2.

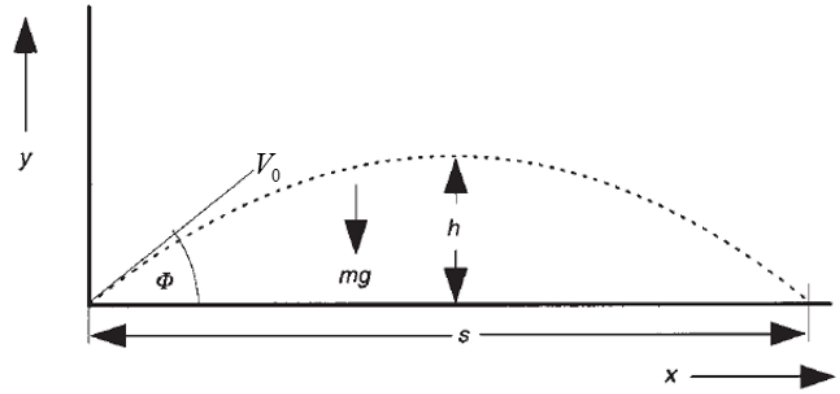

Figure 2 Movement of a mass point under the effect of gravitational force.

Whereas the vertical motion (y axis motion) is under the effect of gravitational acceleration, the horizontal motion (x axis motion) is no under any acceleration motion. It is only under the effect of the x axis component of the velocity which is a constant speed.

There are several formulas which are very important for some calculations. Since in this experiment the main aim is to calculate the **maximum height** and **maximum range**, the formulas of the **maximum height** and **maximum range** should be defined. There are two alternatives in order to find the **maximum height**.

For the first way, the formula of

$$V_{final}^2 = V_{initial}^2 + 2\vec{a}\vec{y} \tag{1}$$

can be used. $V_{initial}$, V_{final}, $\vec{a}$ and $\vec{y}$ are the initial speed, the final speed, the acceleration and the distance, respectively. In this system, $\vec{a}$ and $\vec{y}$ are the gravitational acceleration ($\vec{g}$) and the maximum height($\vec{h}$), respectively. Since the gravitational acceleration is in the opposite direction of the ground, it causes the decrementation of speed. Therefore its sign will be negative. Then the Eq. (1) can be redefined as

$$V_{final}^2 = V_{initial}^2 - 2gh \tag{2}$$

Since the final vertical speed is equal to 0 ($V_{final} = 0$) at the maximum height, Eq. (2) can be arranged as

$$V_{initial}^2 = 2gh \tag{3}$$

where the initial speed can be defined as

$$V_{initial} = V_0 \sin\phi \tag{4}$$

where ϕ is the angle with the horizontal axis x which can also be seen in Fig. 2. After that, the maximum height can be defined as

$$h = \frac{V_0^2 \sin^2\phi}{2g} \tag{5}$$

For the second way, after the ball arrive the highest point, its speed is instantly zero. Then it takes the maximum height h by free motion (without any initial vertical speed). As a consequence, the formula of

$$\vec{y} = \frac{1}{2}\vec{a}t^2 \tag{6}$$

can be used. For this system, $\vec{a}$ and t are the gravitational acceleration ($\vec{g}$) and the half of flying time of the ball, respectively. After that, Eq. (6) can be arranged as

$$\vec{h} = \frac{1}{2}\vec{g}t^2 \tag{7}$$

where the time (t) can be obtained by the general formula of

$$\vec{V} = \vec{a}t \tag{8}$$

which can be redefined as

$$t = \frac{\vec{V}}{\vec{a}}. \tag{9}$$

If the first speed is zero, then the flying time can be found directly by Eq. (9). It is also known that the initial vertical speed is equal to the final vertical speed when the motion is completed. Therefore, the half of flying time can be found by

$$t = \frac{\vec{V}_0 \sin\phi}{\vec{g}}. \tag{10}$$

As a result, Eq. (7) can be arranged as

$$h = \frac{1}{2} g \left(\frac{V_0 \sin\phi}{g} \right)^2 \tag{11}$$

which can be rearranged as

$$h = \frac{V_0^2 \sin^2\phi}{2g} \tag{12}$$

which is also the same result as in Eq. (5). Thus, both Eq. (5) and Eq. (12) can be used for the maximum height calculation.

In this step, after the calculation of the maximum height, the **maximum range** which will be named as s can be found. The horizontal motion has the constant speed which is equal to $V_0 \cos\phi$. The general range formula of the constant speed motion can be expressed as

$$\vec{x} = \vec{V}t \tag{13}$$

which can be used for the calculation of the maximum range s. The half of flying time was found in Eq. (10) which can be used in Eq. (13). Since the motion takes time as double of half of flying time, the whole flying time can be defined as

$$t_{fly} = 2t = 2\frac{\vec{V}_0 \sin\phi}{\vec{g}} \tag{14}$$

where t was found in Eq. (10). After that, Eq. (13) can be written as

$$s = 2V_0 \cos\phi \frac{\vec{V}_0 \sin\phi}{\vec{g}} \tag{15}$$

where $2\sin\phi\cos\phi$ is equal to $\sin 2\phi$. Finally the maximum range s can be found as

$$s = \frac{V_0^2}{g} \sin 2\phi \tag{16}$$

4. Procedure

• The maximum height h and maximum range s will be investigated for the different motion angles.

• Please only use the first speed level of the projection system due to the laboratory conditions that cannot especially satisfy the maximum range s.

a) Using the measured velocity from the system’s digital segment, measure and calculate the maximum height h and maximum range s for the angle of 15° and fill in the Table 1 below. Please also show your calculations clearly in the part calculations.

V_0 (Measured)		Measured	Calculated
	Max. height h		
	Max. range s		

Table 1

Calculations:

b) Using the measured velocity from the system's digital segment, measure and calculate the maximum height h and maximum range s for the angle of 30° and fill in the Table 2 below. Please also show your calculations clearly in the part calculations below.

V_0 (Measured)		Measured	Calculated
	Max. height h		
	Max. range s		

Table 2

Calculations:

c) Using the measured velocity from the system's digital segment, measure and calculate the maximum height h and maximum range s for the angle of 45° and fill in the Table 3 below. Please also show your calculations clearly in the part calculations below.

V_0 (Measured)		Measured	Calculated
	Max. height h		
	Max. range s		

Table 3

Calculations:

d) Using the measured velocity from the system's digital segment, measure and calculate the maximum height h and maximum range s for the angle of 60° and fill in the Table 4 below. Please also show your calculations clearly in the part calculations below.

V_0 (Measured)		Measured	Calculated
	Max. height h		
	Max. range s		

Table 4

Calculations:

e) Using the measured velocity from the system's digital segment, measure and calculate the maximum height h and maximum range s for the angle of 75° and fill in the Table 5 below. Please also show your calculations clearly in the part calculations below.

V_0 (Measured)		Measured	Calculated
	Max. height h		
	Max. range s		

Table 5

Calculations:

5. Results and Discussions

Discuss the results