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ESE – 40… ENERGY SYSTEMS LABORATORY -…
Heat Conduction Experiment Report

Student No :
Name-Surname :
Experiment Date :
Grade :

2.3 Heat Conduction Experiment

2.3.1 Objective

The purpose of this experiment is to determine the constant of proportionality (the thermal conductivity *k*) for one-dimensional steady flow of heat, to understand the use of the Fourier's law in determining heat rate through solids, and to demonstrate the effect of contact resistance on thermal conduction between adjacent materials.

2.3.2 Introduction

Thermal conduction is the transfer of heat energy in a material due to the temperature gradient within it. It always takes place from a region of higher temperature to a region of lower temperature. A solid is chosen for the experiment of pure conduction because both liquids and gasses exhibit excessive convective heat transfer. For practical situation, heat conduction occurs in three dimensions, a complexity which often requires extensive computation to analyze. For experiment, a single dimensional approach is required to demonstrate the basic law that relates rate of heat flow to temperature gradient and area.

2.3.3 Theory

2.4.3.1 Linear Heat Conduction

According to Fourier's law of heat conduction: If a plane wall of thickness (ΔL) and area (A) supports a temperature difference (ΔT) then the heat transfer rate per unit time (Q) by conduction through the wall is found as shown in the following formulas.

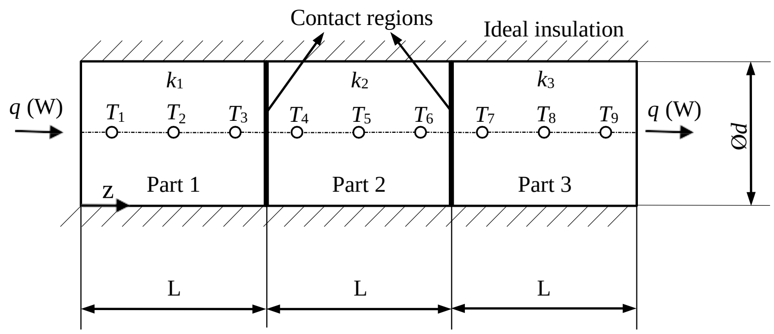


Figure 2.4.1. The Schematic View of Linear Heat Conduction Experiment Setup

The steady-state heat conduction equation in 1-D Cartesian coordinates is

$$\frac{d}{dz}\left(k\frac{dT}{dz}\right) = 0 \tag{2.4.1}$$

Integrate from the left boundary *z* = 0, to some arbitrary location *z* less than the bar length *L*

$$\int_0^z \frac{d}{dz}\left(k\frac{dT}{dz}\right) dz = 0 \tag{2.4.2}$$

$$k\left.\frac{dT}{dz}\right|_z - k\left.\frac{dT}{dz}\right|_0 = 0 \tag{2.4.3}$$

At z = 0, the heat flux is known:

$$q"(0) = \frac{q}{A_0} = -k\left.\frac{dT}{dz}\right|_0 \tag{2.4.4}$$

Where $A_0 = \frac{\pi d^2}{4}$, d is the diameter of the bar and q is the input power as you can get them experiment setup's schematic view in Fig.2.4.1. Upon substitution, Equation 2.4.3 becomes:

$$k\left.\frac{dT}{dz}\right|_z + \frac{q}{A_0} = 0 \tag{2.4.5}$$

Integrate between any two thermocouples, e.g., from z_i to z_{i+1}

$$\int_{T_i}^{T_{i+1}} kdT = -\int_{z_i}^{z_{i+1}} \frac{q}{A_0} dz \tag{2.4.6}$$

Assuming the thermal conductivity *(k)* is constant between each thermocouple position, and the cross-sectional area is the same, the temperature at any thermocouple location can be related to the temperature at any other thermocouple location by

$$T_{i+1} - T_i = -\frac{q}{k_i A_0}(z_{i+1} - z_i) \tag{2.4.7}$$

or solving for the thermal conductivity

$$k = -\frac{q}{A_0}\frac{z_{i+1} - z_i}{T_{i+1} - T_i} \tag{2.4.8}$$

Equation 2.4.8 may be used to estimate the local thermal conductivity. Linear heat conduction experiment setup can be seen in Fig.2.4.2.

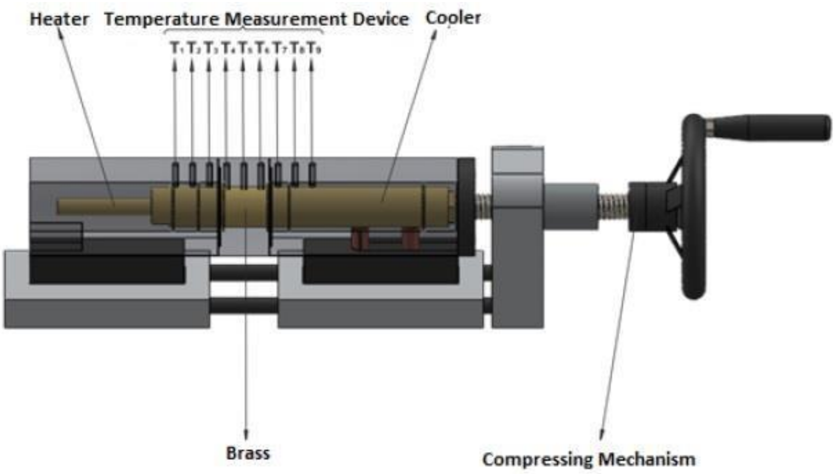


Figure 2.4.2. Linear Heat Conduction Experiment Setup

2.4.3.1.1 Contact Resistance

In the absence of good thermal contact, the temperature distribution will show a drop at the interface between any two sections. However, the heat flux will be the same through both materials at steady-state. We can define the temperature drop across the interface in terms of a *contact resistance* such that:

$$R" = \frac{\Delta T_{gap}}{q"} \tag{2.4.9}$$

In this experiment however, the temperatures are not measured at the interface, and the gap conductance, as well as the thermal conductivity must be inferred from the measured temperature distribution. Consider the diagram of the interface illustrated below, where T_1 and T_2 refer to thermocouple locations on either side of the interface.

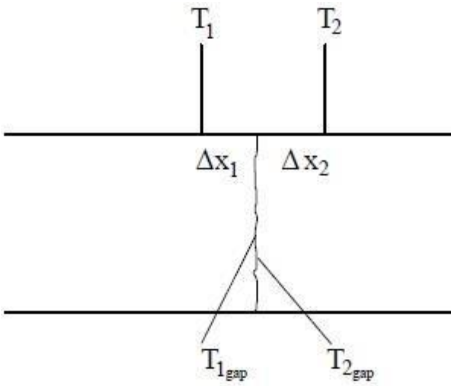


Figure 2.4.3. Contact Regions View

In addition, we define $T_{1,gap}$ as the temperature at location $z_{1,gap}^{(-)}$ on the left-hand surface of the interface and $T_{2,gap}$ as the temperature at locationon $z_{1,gap}^{(+)}$ on the right hand surface of the interface. The temperature distribution prior to the interface is obtained from the steady-state heat conduction equation in 1-D Cartesian geometry:

$$\frac{d}{dz}\left(k\frac{dT}{dz}\right) = 0 \tag{2.4.10}$$

Integrate from the first thermocouple location z = z_0, to some arbitrary location z less than $z_{1,gap}$.

$$\int_{z_0}^{z} \frac{d}{dz}\left(k\frac{dT}{dz}\right) dz = 0 \tag{2.4.11}$$

$$k\frac{dT}{dz}\bigg|_z - k\frac{dT}{dz}\bigg|_0 = 0 \tag{2.4.12}$$

Since the cross-sectional area is constant, at steady state the heat flux is known:

$$q"(z_0) = q"(0) = \frac{q}{A_0} = -k\left.\frac{dT}{dz}\right|_{z_0} \tag{2.4.13}$$

Upon substitution, Equation 2.4.12 becomes:

$$k\left.\frac{dT}{dz}\right|_{z} + \frac{q}{A_0} = 0 \tag{2.4.14}$$

Integrate from z=z_0 to any arbitrary thermocouple location prior to the interface

$$\int_{T_0}^{T_n} kdT = -\int_{z_0}^{z_n} \frac{q}{A_0} dz \tag{2.4.15}$$

Assuming the thermal conductivity is constant, the temperature at any thermocouple location prior to the interface is

$$T_n - T_0 = -\frac{q}{k_i A_0}(z_n - z_0) \tag{2.4.16}$$

where T_0 is the first thermocouple location. The temperature on the left-hand side of the interface is then simply

$$T_{1,gap} - T_0 = -\frac{q}{k_i A_0}(z_{1,gap} - z_0) \tag{2.4.17}$$

The temperature drop across the interface is written in terms of the gap conductance as

$$T_{1,gap} - T_{2,gap} = -\frac{q}{A_0}R" \tag{2.4.18}$$

The temperature distribution after the interface is again obtained by integrating the conduction equation from $z = z^{(+)}_{1,gap}$, to some arbitrary location z less than $z_{2,gap}$, where $z_{2,gap}$ is the second interface location.

$$\int_{z^{(+)}_{1,gap}}^{z} \frac{d}{dz}\left(k\frac{dT}{dz}\right) dz = 0 \tag{2.4.19}$$

$$k\left.\frac{dT}{dz}\right|_{z} - k\left.\frac{dT}{dz}\right|_{z^{(+)}_{1,gap}} = 0 \tag{2.4.20}$$

Since the heat flux across the interface is unchanged:

$$-k\left.\frac{dT}{dz}\right|_{z^{(+)}_{1,gap}} = \frac{q}{A_0} \tag{2.4.21}$$

Upon substitution, Equation2.4.20 becomes:

$$k\left.\frac{dT}{dz}\right|_z + \frac{q}{A_0} = 0 \tag{2.4.22}$$

$$\int_{T_{1,gap}}^{T} kdT = -\int_{z_{1,gap}^{(+)}}^{z} \frac{q}{A_0} dz \tag{2.4.23}$$

Assuming the thermal conductivity is constant, the temperature drop following the interface is

$$T - T_{2,gap} = -\frac{q}{k_i A_0}(z - z_{1,gap}) \tag{2.4.24}$$

We can eliminate the temperature at the gap interface, and write the temperature at any point after the interface by adding equations 2.4.17, 2.4.18 and 2.4.24 to give

$$T - T_0 = -\frac{q}{A_0}\left(\frac{z - z_0}{k} + R"\right) \tag{2.4.25}$$

The analysis can be repeated for the second interface, such that the temperature at any thermocouple location can be written as

Prior to interface 1($z_n < z_{1,gap}$)

$$T_n = T_0 - \frac{q}{k_i A_0}(z_n - z_0) \tag{2.4.26}$$

Following interface 1, and prior to interface 2($z_{2,gap} < z_n < z_{2,gap}$)

$$T_n = T_0 - \frac{q}{A_0}\left(\frac{z - z_0}{k} + R"_1\right) \tag{2.4.27}$$

Following interface 2($z_{2,gap} < z_n$)

$$T_n = T_0 - \frac{q}{A_0}\left(\frac{z - z_0}{k} + R"_1 + R"_2\right) \tag{2.4.28}$$

2.4.3.2 Radial Heat Conduction

When the inner and outer surfaces of a thick-walled cylinder are each at a different uniform temperature, heat flows radially through the cylinder wall. The disk can be considered to be constructed as a series of successive layers. From continuity considerations the radial heat flow through each of the successive layers in the wall must be constant if the flow is steady but since the area of the successive layers increases with radius, the temperature gradient must decrease with radius.

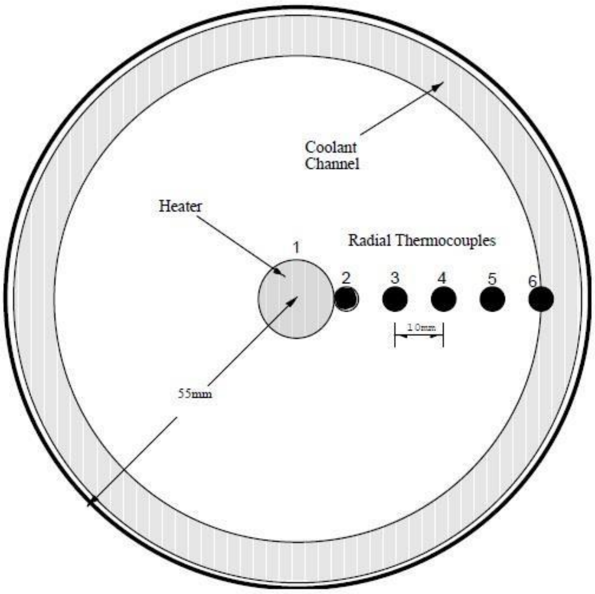


Figure 2.4.4. The Schematic View of Radiation Heat Conduction Experiment Setup

The steady-state heat conduction equation in 1-D cylindrical geometry is

$$\frac{1}{r}\frac{d}{dr}\left(rk\frac{dT}{dr}\right) = 0 \tag{2.4.29}$$

Multiply through by r and integrate from the heater radius r_H to some arbitrary radius r less than the outer radius, r_o

$$\int_{r_H}^{r}\frac{d}{dr}\left(rk\frac{dT}{dr}\right)dr = 0 \tag{2.4.30}$$

$$rk\frac{dT}{dr}\bigg|_{r} - rk\frac{dT}{dr}\bigg|_{r_H} = 0 \tag{2.4.31}$$

At r_H, the heat flux is known:

$$q''(r_H) = \frac{q}{A_H} = -k\frac{dT}{dr}\bigg|_{r_H} \tag{2.4.32}$$

where $A_H = 2\pi r_H L$, L is the thickness of the disk and q is the input power to the heater. Upon substitution, Equation 2.4.31 becomes:

$$rk\left.\frac{dT}{dr}\right|_r + \frac{q}{2\pi L} = 0 \tag{2.4.33}$$

Divide by r and integrate between two adjacent thermocouples (e.g., from r_i to r_{i+1})

$$\int_{T_i}^{T_{i+1}} k\, dT = -\int_{r_i}^{r_{i+1}} \frac{q}{2\pi L}\frac{dr}{r} \tag{2.4.34}$$

Assuming the thermal conductivity, k, is constant, the temperature at any thermocouplelocation can be related to the temperature at any other thermocouple location by

$$T_{i+1} - T_i = -\frac{q}{2\pi kL}\ln\left(\frac{r_{i+1}}{r_i}\right) \tag{2.4.35}$$

or solving for the thermal conductivity

$$k = -\frac{q}{2\pi L}\frac{1}{T_{i+1} - T_i}\ln\left(\frac{r_{i+1}}{r_i}\right) \tag{2.4.36}$$

If we let $r_i = r_H$, then Equation 2.4.35 can be used to relate the temperature at any thermocouple location to the temperature at the heater surface by

$$T_n = T(r_H) - \frac{q}{2\pi kL}\ln\left(\frac{r_n}{r_H}\right) \tag{2.4.37}$$

Equation 2.4.36 may be used to estimate the local thermal conductivity in the radial experiment. Radial heat conduction experiment setup can be seen in Fig.2.4.5.

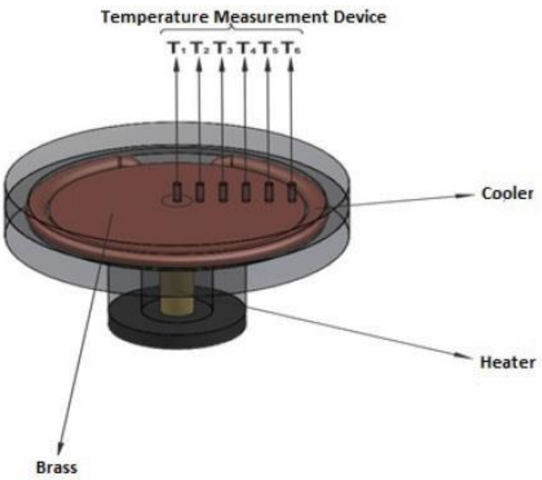


Figure 2.4.5. Radial Heat Conduction Experiment Setup

2.4.4. Experiments
2.4.4.1. Linear Heat Conduction

Aim of the Experiment: To comprehend how to calculate thermal conductivity (*k*).

The necessary data for calculations will be recorded to the table given below.

Material:									
Power(W)	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9
Distance from the origin	0,015	0,030	0,045	0,060	0,075	0,090	0,105	0,120	0,135

Calculations: Using the equation given below, calculate the thermal conductivity.

Thermal conductivity is defined as:

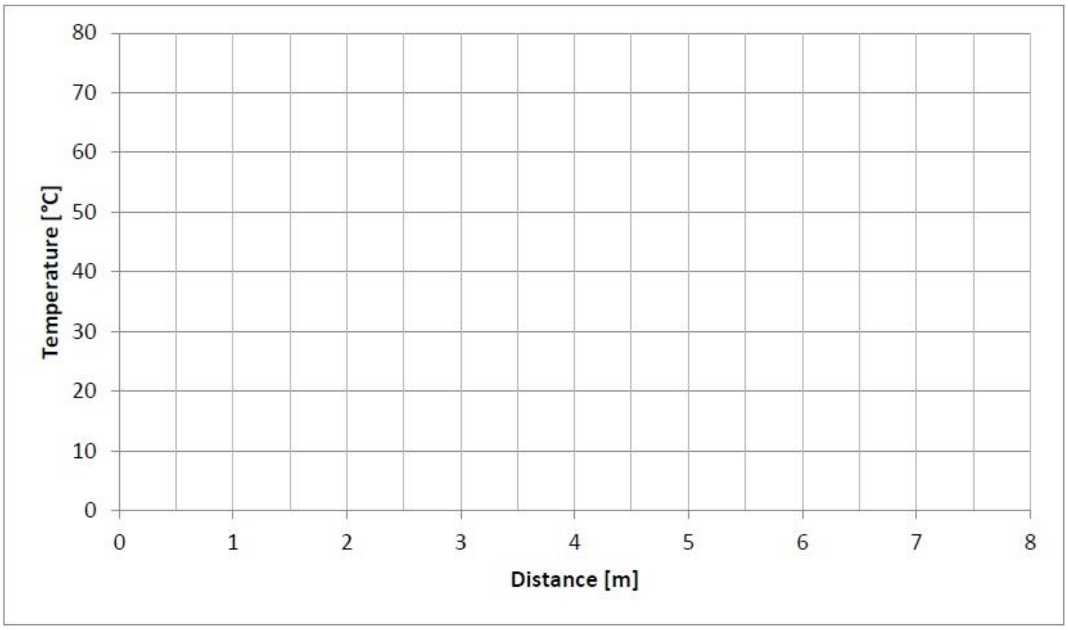
$$k = \frac{q \times \Delta L}{A \times \Delta T}$$

ΔL=1,5cm=0,015m

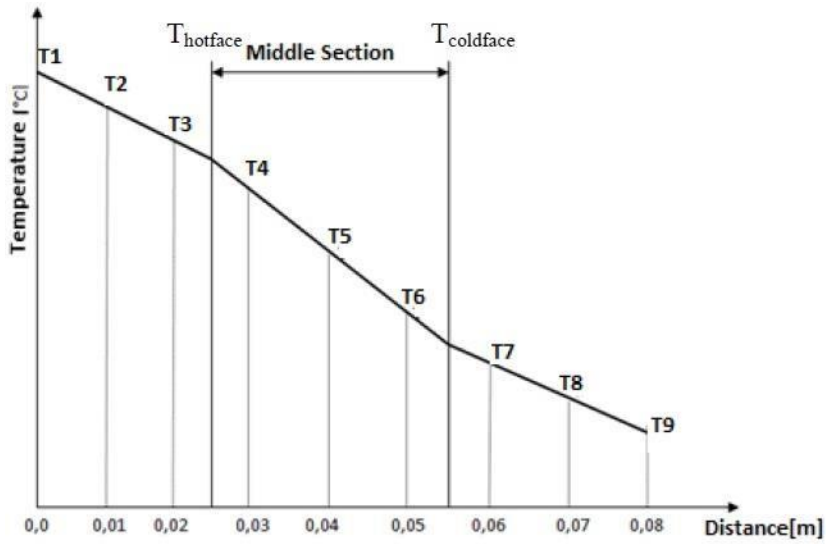
where:

A=7,065x10^{-4} m^2

Plot a graph of temperature against position along the bar and draw the best straight line through the points. Comment on the graph.



A sample graph of temperature against position along the bar can be seen.



Compare your result with Table 2.4.1.

Table 2.4.1. Thermal Conductivities for Different Material Types

Materials in Normal Conditions (298 K, 24.85 °C)		Thermal Conductivity (k) W/m°C
Metals	Pure Aluminium	205-237
	Aluminium Alloy (6082)	170
	Brass (CZ 121)	123
	Brass (63% Copper)	125
	Brass (70% Copper)	109-121
	Pure Copper	353-386
	Copper (C101)	388
	Light Steel	50
	Stainless Steel	16
Gas	Air	0.0234
	Hydrogen	0.172
Others	Asbestos	0.28
	Glass	0.8
	Water	0.6
	Wood	0.07-0.2

2.4.4.1 Radial Heat Conduction

Aim of the Experiment: To comprehend how to calculate thermal conductivity (*k*).

The necessary data for calculations will be recorded to the table given below.

Material:						
Power(W)	T_1	T_2	T_3	T_4	T_5	T_6
Brass						
Radial Distance from the origin	0,015	0,030	0,045	0,060	0,075	0,090

Calculations: Using the equation given below, calculate the thermal conductivity.

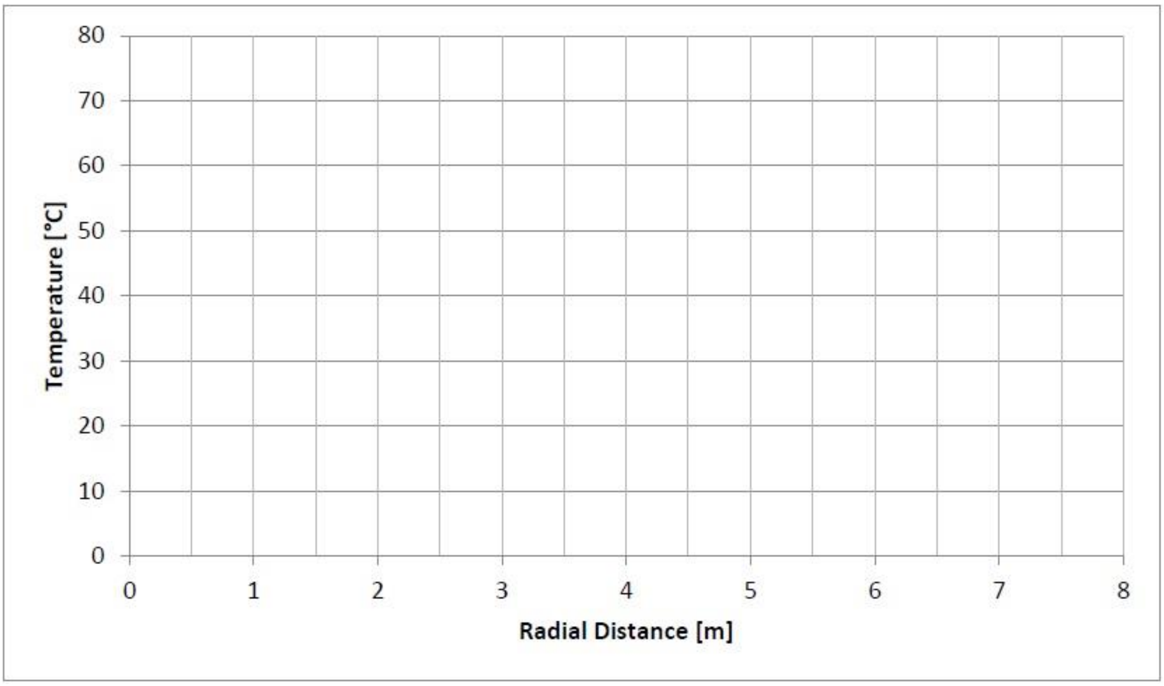
Thermal conductivity is defined as:

$$k = \frac{q\ln(\frac{Ra}{Rb})}{2\pi L(Tb-Ta)}$$

where:

L=12cm=0,012m

Plot a graph of temperature against position along the bar and draw the best straight line through the points. Comment on the graph.



2.3.4 Report

In your laboratory reports must have the followings;

a) Cover.
b) All the necessary calculations using measured data.
c) Conclucisions.

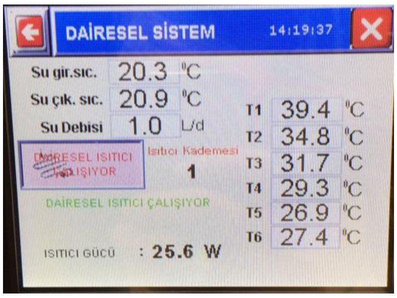
d)

Brass (Pirinç)

Copper (Bakır)

Aluminum (Alüminyum)

Steel (Çelik):



Radial (Brass) (Radyal Pirinç):